

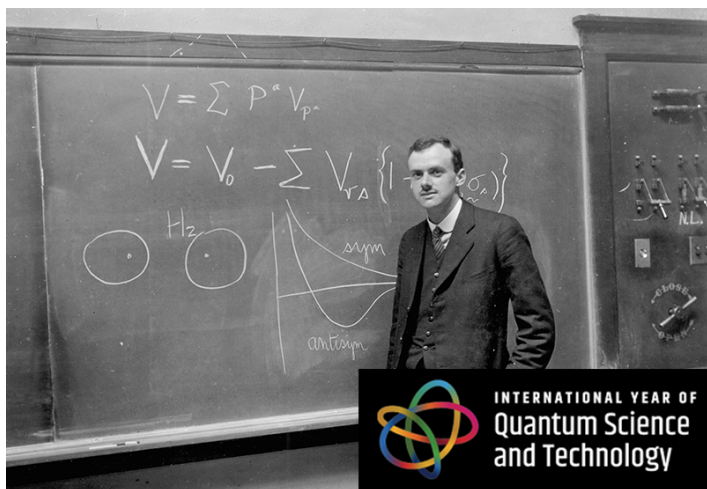
Quantum Milestones, 1928: The Dirac Equation Unifies Quantum Mechanics and Special Relativity

A seminal paper by Paul Dirac, who relied on mathematical intuition, laid the foundation for quantum electrodynamics.

By **Nyla Husain**

For the *International Year of Quantum Science and Technology*, we are republishing stories on the history of quantum physics from the archives of *Physics Magazine* and *APS News*. The *original version of this story* was published in *APS News* on November 19, 2024.

In the first weeks of 1928, “The Quantum Theory of the Electron,” a paper authored by a young British physicist named Paul Dirac, took the theoretical physics community by storm [1].



Paul Dirac once said that his theories are “built up from physical concepts which cannot be explained in words at all.”

Credit: AIP Emilio Segrè Visual Archives

The 25-year-old Dirac had worked alone to develop his theory in the last months of 1927. It was the first to fully incorporate the framework for quantum mechanics—formulated over the previous two years by the likes of Werner Heisenberg and Erwin Schrödinger—with Albert Einstein’s special relativity. Many of his peers, including Schrödinger, on whose wave equation his results were based, had tried and failed to do the same thing. Dirac’s results would soon transform the way theoretical physicists understood how light and matter interact—and, by accident, uncover a completely new class of particles known as antimatter.

When Dirac entered the field of quantum theory, he came with an unorthodox skill set. With bachelor’s degrees in electrical engineering and applied mathematics from the University of Bristol, UK, he hadn’t obtained a formal physics education.

“It made him a very unusual animal,” says Graham Farmelo, biographer, science writer, and author of *The Strangest Man: The Hidden Life of Paul Dirac, Mystic of the Atom*. “Wherever he went, he was an outsider.” Being an outsider, it seemed, allowed Dirac to approach the same questions with a different lens than his peers—that of both an engineer trained in practical thinking and a gifted mathematician who could see equations in terms of pictures and diagrams.

Dirac’s trajectory into theoretical physics arguably began with his passion for Albert Einstein’s general theory of relativity. After the Eddington solar eclipse experiment confirmed Einstein’s

theory in 1919, Dirac attended a series of lectures at Bristol given by Charlie Broad, a philosopher of science from Cambridge University. Dirac was no philosopher, but Broad's talks on relativity and scientific thought clarified the theory and inspired him. Farmelo writes in his book that Dirac made a hobby of producing relativistic versions of Newtonian theories "like an engineer upgrading tried-and-tested designs to ones that perform to a higher specification."

In 1923, when Dirac became a doctoral student at St. John's College in Cambridge, UK, he hoped to study relativity. Instead, he was assigned to Ralph Fowler, one of few researchers in Britain involved in the burgeoning field of quantum theory.

Two years into Dirac's studies, German physicists Werner Heisenberg, Max Born, and Pascual Jordan revolutionized the old quantum theory by introducing a new approach to describe particle behavior in atomic systems. Their matrix mechanics was the first mathematical formulation to describe, with arrays of discrete numbers, only *observable* quantities of a particle's behavior—ones that could be measured experimentally—like its position or momentum. Months later, Schrödinger—inspired by Louis de Broglie's idea that matter behaves like a wave—proposed an entirely different, but mathematically equivalent, formulation of particle behavior based on the better-known mathematics of waves.

This reinterpretation overcame critical limitations of the old quantum theory. Yet, by late 1927, the theory was far from complete. Among its most glaring deficiencies was the fact that, despite multiple efforts, no equation adequately accounted for relativistic effects—changes in particles' behavior when moving at incredibly high speeds. Dissatisfied with previous attempts, Dirac set out to solve the mathematical puzzle himself.

Dirac made a habit of working independently and preferred to spend most of his time alone. Described by colleagues as reticent and unemotional, he developed a reputation for monosyllabic responses—"yes," "no," or "I don't know." His writing was equally precise and condensed, saying only what needed to be said to convey his ideas.

"Dirac would give a seminar and when asked a question, he would just repeat verbatim what he said the first time," says David Kaiser, physicist and historian at MIT. "Not to be rude, I



From left to right, Paul Dirac, Werner Heisenberg, and Erwin Schrödinger at a Stockholm train station in 1933.

Credit: Max Planck Institute; courtesy AIP Emilio Segrè Visual Archives

don't think, but because he thought that was the most economical expression."

By January 2, 1928, Dirac had written up his results and submitted them to the *Proceedings of the Royal Society A*. His formulation—a fully relativistic version of Schrödinger's wave equation for the electron—could solve, in sharp detail, the spectral character of emission or absorption of radiation by an atom, which Schrödinger's equation had failed to do.

More surprising results unfurled when Dirac extended his equation to describe an electron interacting with an electromagnetic field. Experimentalists had confirmed that the electron's intrinsic angular momentum, or spin, was equal to $1/2$, but theoreticians couldn't figure out how to properly incorporate it into their theories. With his new equation, Dirac had found, almost as an afterthought, that the spin emerged naturally. His colleagues were shocked and energized by these results. "It was a happy surprise to see that spin was already kind of built into his equation," Kaiser says.

The Dirac equation was simple and elegant, yet dense with implications. Perhaps its most profound feature was that,

instead of producing two components for negative and positive spin states, it produced four: a negative and positive spin state for each of two particles with positive and negative energy states. It was absurd to think that an electron could possess energy of less than zero, but the alternative was even stranger: that there existed an entirely new, unobserved particle. “Dirac eventually came to convince himself that these would correspond to objects with the opposite electric charge,” Kaiser says.

It wasn’t until 1932 that American physicist Carl Anderson confirmed the existence of the antielectron, or positron as he called it, with cosmic-ray experiments. Dirac would earn the 1933 Nobel Prize in Physics for his discovery, which he shared with Schrödinger.

The Dirac equation laid the foundation for quantum

electrodynamics, a quantum field theory that has enabled technologies like lasers and semiconductors. Yet the technique employed to make the theory useful—renormalization—repulsed Dirac because he found it mathematically ugly. Positron emission tomography (PET) scanning, first developed in the 1960s, also wouldn’t have been possible without his discovery, but it’s unclear whether he knew about the medical imaging technique or would have cared. Dirac generally followed his intuition—and his intuition spoke only in equations.

Nyla Husain is the science communications manager at APS.

REFERENCES

1. P. A. M. Dirac, “The quantum theory of the electron,” *Proc. R. Soc. Lond. A* **117**, 610 (1928).